Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

4714309059

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2021**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

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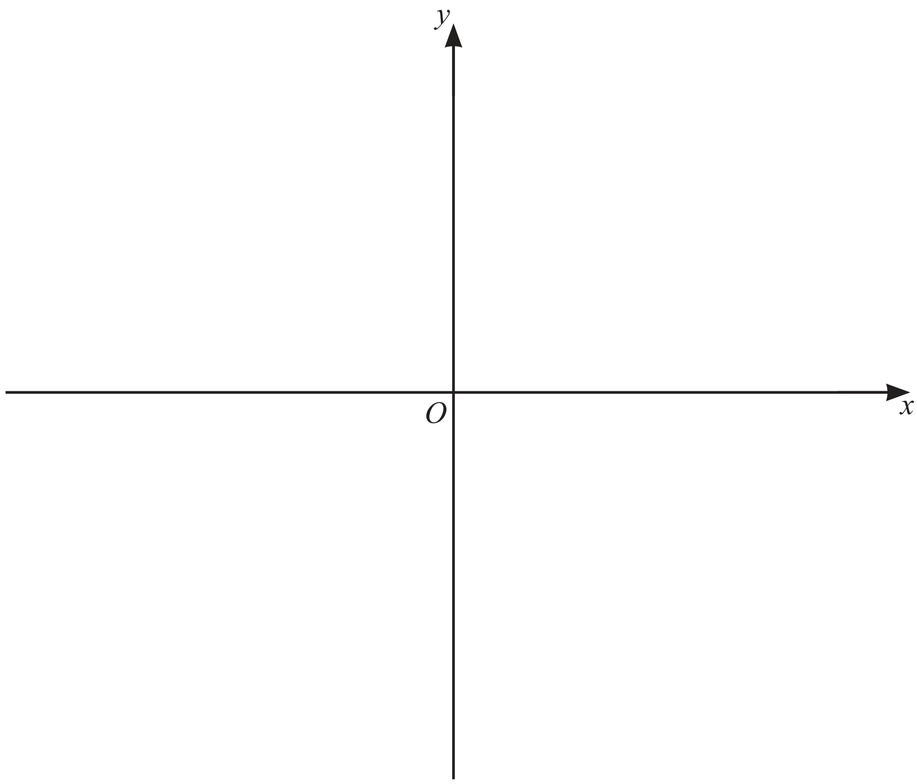
1 Using the binomial theorem, expand $(1+e^{2x})^4$, simplifying each term. [2]

2 On the axes, sketch the graph of $y = 3(x-3)(x-1)(x+2)$ stating the intercepts with the coordinate axes. [3]



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3 Find the values of the constant k for which $(2k-1)x^2+6x+k+1=0$ has real roots. [5]

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4 The polynomial $p(x) = mx^3 - 29x^2 + 39x + n$, where m and n are constants, has a factor $3x - 1$, and remainder 6 when divided by $x - 1$. Show that $x - 2$ is a factor of $p(x)$. [6]

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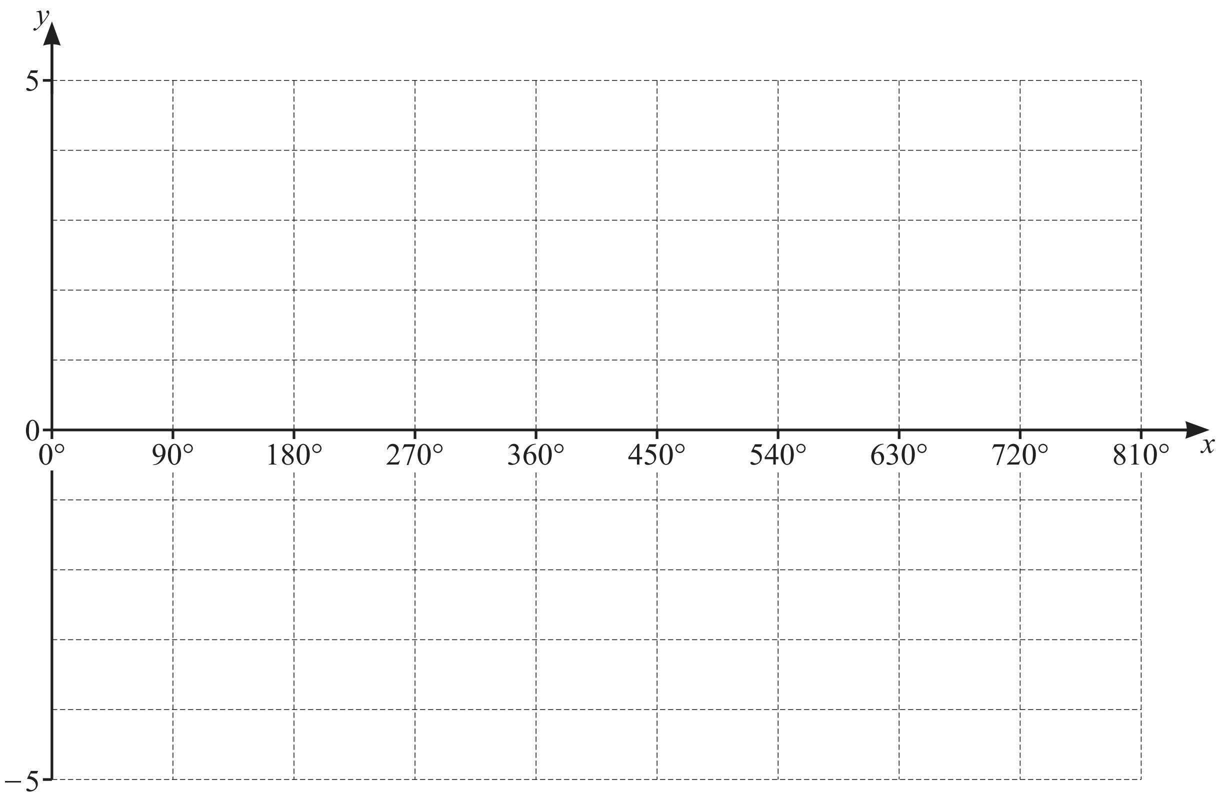
5 The function f is defined, for $0° \leqslant x \leqslant 810°$, by $f(x) = -2 + \cos\frac{2x}{3}$.

(a) Write down the amplitude of f. [1]

(b) Find the period of f. [2]

(c) On the axes, sketch the graph of $y = f(x)$. [2]



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6 The points $A(5, -4)$ and $C(11, 6)$ are such that AC is the diagonal of a square, $ABCD$.

(a) Find the length of the line AC. [2]

(b) **(i)** The coordinates of the centre, E, of the square are $(8, y)$. Find the value of y. [1]

(ii) Find the equation of the diagonal BD. [3]

(iii) Given that the x-coordinate of B is less than the x-coordinate of D, write $\overrightarrow{EB}$ and $\overrightarrow{ED}$ as column vectors. [2]

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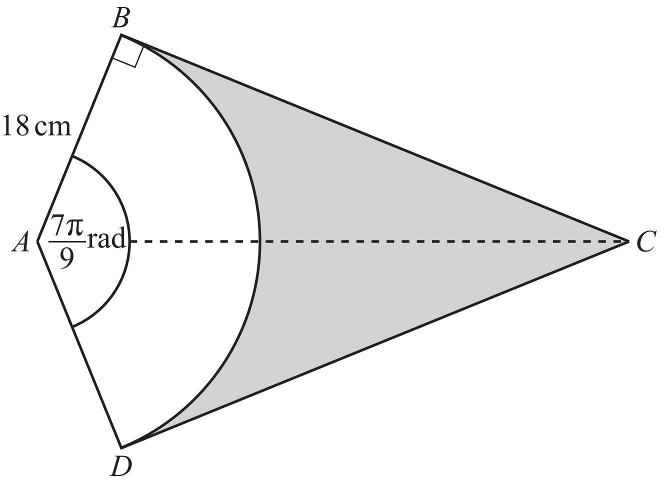

DAB is a sector of a circle, centre *A*, radius 18 cm. The lines *CB* and *CD* are tangents to the circle. Angle *DAB* is $\frac{7\pi}{9}$ radians.

(a) Find the perimeter of the shaded region. [3]

(b) Find the area of the shaded region. [3]

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8 A particle moves in a straight line so that, t seconds after passing through a fixed point O, its velocity, $v\,\mathrm{ms}^{-1}$, is given by $v = 3t^2 - 30t + 72$.

(a) Find the distance between the particle's two positions of instantaneous rest. [6]

(b) Find the acceleration of the particle when $t = 2$. [2]

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9 Solve the following simultaneous equations.

$$4x^2 + 3xy + y^2 = 8$$
$$xy + 4 = 0$$

[6]

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10 **(a)** Find $\int (e^{x+1})^3 dx$. [2]

(b) **(i)** Differentiate, with respect to x, $y = x \sin 4x$. [2]

(ii) Hence show that $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4x \cos 4x dx = \frac{1}{8} - \frac{\pi\sqrt{3}}{6}$. [4]

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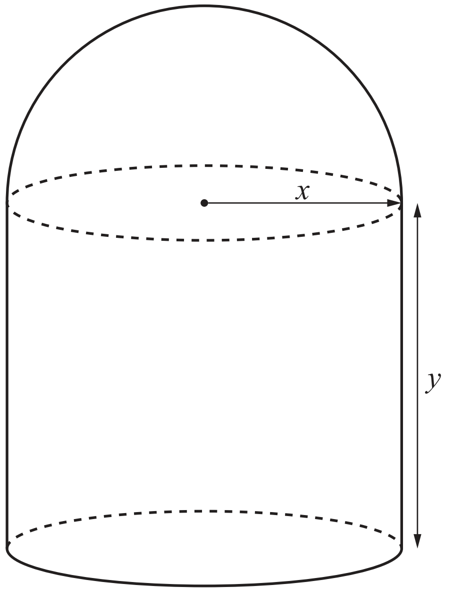
11 In this question all lengths are in centimetres.

The volume and surface area of a sphere of radius r are $\frac{4}{3}\pi r^3$ and $4\pi r^2$ respectively.



The diagram shows a solid object made from a hemisphere of radius x and a cylinder of radius x and height y. The volume of the object is $500\,\text{cm}^3$.

(a) Find an expression for y in terms of x and show that the surface area, S, of the object is given by $S = \frac{5}{3}\pi x^2 + \frac{1000}{x}$. [4]

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(b) Given that x can vary and that S has a minimum value, find the value of x for which S is a minimum. [4]

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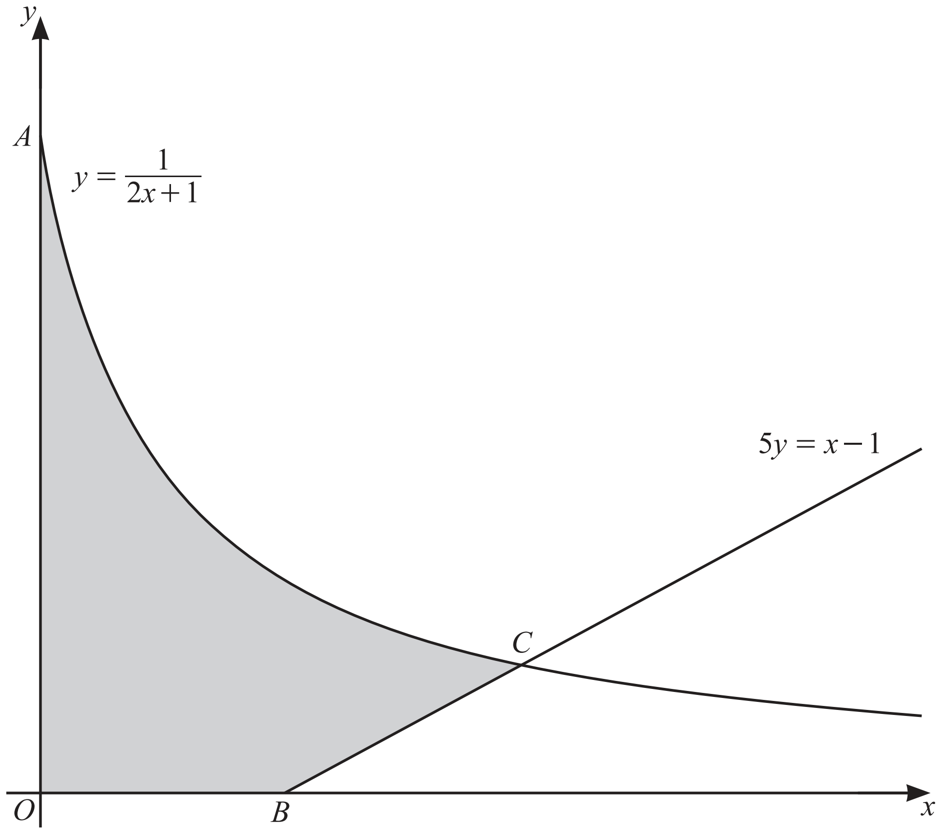
12 DO NOT USE A CALCULATOR IN THIS QUESTION.



The diagram shows part of the curve $y = \frac{1}{2x+1}$ and part of the line $5y = x - 1$.

The curve meets the y-axis at point A. The line meets the x-axis at point B. The line and curve intersect at point C.

(a) (i) Find the coordinates of A and B. [1]

(ii) Verify that the x-coordinate of C is 2. [2]

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(b) Find the exact area of the shaded region. [5]

Question 13 is printed on the next page.

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13 The functions f and g are defined, for $x > 0$, by

$$f(x) = \frac{2x^2 - 1}{3x},$$

$$g(x) = \frac{1}{x}.$$

(a) Find and simplify an expression for $fg(x)$. [2]

(b) **(i)** Given that f^{-1} exists, write down the range of f^{-1}. [1]

(ii) Show that $f^{-1}(x) = \frac{px + \sqrt{qx^2 + r}}{4}$, where p, q and r are integers. [4]

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